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Are There Other Types of Skyrmions?

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Due to the successes of the Skyrme model, we explore the possibility that other types of skyrmions exist, due solely to the existence of a pseudoscalar triplet with a Lagrangian of the Skyrme form. The physical meaning of other types of skyrmions is discussed.

Skyrme (1961) first suggested that baryons are solitons in the nonlinear sigma model. In the large-N limit QCD becomes equivalent to an effective field theory of mesons (t'Hooft, 1974), which reduces to a nonlinear sigma model in the low energy limit. Baryons may emerge as solitons in this theory (Whitten, 1979, 1983). Accordingly, Adkins *et al.* (1983) and other authors calculated the static properties and scattering of baryons and others, obtaining good results. The above suggest some questions about the skyrmion which go beyond the horizons encompassed by nucleon properties.

Pions and σ mesons, through a nonlinear interaction, can form a soliton, which may be interpreted as a baryon in the Skyrme model. This soliton is called a skyrmion. The Lagrangian of the Skyrme model is (Adkins *et al.*, 1983)

$$\mathscr{L} = \frac{1}{16} f^2 \operatorname{Tr}(\partial_\mu U \cdot \partial^\mu U^\dagger) + \frac{1}{32e^2} \operatorname{Tr}[(\partial_\mu U) U^\dagger, (\partial_\nu U) U^\dagger]^2 \tag{1}$$

where f is the decay constant of the pion, e is a dimensionless parameter, $U = \exp[(2i/f)\boldsymbol{\tau} \cdot \boldsymbol{\pi}]$, and $\boldsymbol{\pi}$ is the pseudoscalar triplet of the pion field.

Let us view Skyrme's Lagrangian (1) from a different angle. If we abstract the physical meaning of the field and parameters from (1) and consider (1) as a mathematical formula, then $\boldsymbol{\pi}$ can be not only a pion field, but also an arbitrary pseudoscalar triplet field. The interactions of the arbitrary pseudoscalar triplet may not be as strong as those of the pion.

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The formation of the soliton does not depend on the strength of the interaction (Lee, 1981), but depends only on the character of the nonlinear interaction. Now the problem is whether there exist other pesudoscalar triplets besides the pion, and if one exists, whether it makes up a Skyrme Lagrangian (1).

Let us look for another pseudoscalar triplet. There exists only one physical Higgs scalar in the minimal standard electroweak model (MSM) (Weinberg, 1967; Salam, 1968) after the other three components of the Higgs doublet are eaten by $W^{\pm}$, Z^0. However, there are several questions concerning MSM: (1) Yukawa couplings differ from each other by at least five orders of magnitudes (Harari, 1987) (if neutrinos are massless) and possibly by nine of more orders of magnitude (if neutrinos have small Dirac masses). This is very unattractive. (2) The measurements of a long B lifetime ($1.11 \pm 0.16 \times 10^{-12}$ sec) and a relatively small top mass (40–50 GeV) severely constrain the parameters in the Kobayashi-Maskawa (KM) matrix, even though the theoretical uncertainties are at present too large to conclude that there exists a serious problem (Bertolini, 1986). (3) Peccei-Quinn symmetry (Peccei and Quinn, 1977a,b) requires at least two Higgs doublets in the standard model. (4) In the supersymmetric extension (Llewellyn Smith, 1984) of the standard model at least two Higgs doublets are needed.

We will consider an extension of MSM. Because only Higgs doublets can couple to fermions and give their masses, a minimal extension of the MSM consists in adding a second $SU(2)_L$ doublet to the Higgs sector. There are several advantages for the two-Higgs-doublet model: (1) If there are only Higgs doublets, then $\rho = M_W^2/(M_Z^2 \cos^2\theta_W) = 1$ automatically (Jarlskog, 1985). (2) Two different vacuum expectation values (VEV) are available to give masses to the vector bosons and fermions, or the fermion mass scale is determined by the VEV of two Higgs rather than determined only by widely differing Higgs coupling (Haber *et al.*, 1979). (3) There exist charged Higgs bosons which are easier to detect experimentally.

Two Higgs doublets have eight degrees of freedom. There will be three Goldstone bosons to give masses to the $W^{\pm}$ and Z^0; thus, five physical Higgs particles will remain. There may be a pseudoscalar Higgs triplet in the five physical Higgs. Haber *et al.* (1979) (HKS) have discussed a two-Higgs-doublet model which is not yet excluded by current data. If the mass difference of different components in the same doublet and Cabibbo angles may be neglected, then equation (6) of the HKS model may be written

$$\begin{aligned}\mathcal{L} = \frac{-mg}{2\beta m_W}(&\bar{d}d\phi \sin\alpha + \bar{d}dh^0 \cos\alpha - i\,\bar{d}\gamma_5\, dH^0 \cos\beta \\ &+ \bar{u}u\phi \sin\alpha + \bar{u}uh^0 \cos\alpha + i\bar{u}\gamma_5 uH^0 \cos\beta \\ &+ \sqrt{2}\bar{u}\gamma_5\, dH^+ - \sqrt{2}\,\bar{d}\gamma_5 uH^{-1})\end{aligned} \tag{2}$$

where ϕ, h^0, H^0, and $H^{\pm}$ are five physical Higgs, and m is the mass of u or d quark. It is obvious that H^0, $H^{\pm}$ may construct a pesudoscalar Higgs triplet $\mathbf{H}$.

In the case there exists a pseudoscalar Higgs triplet $\mathbf{H}$, the Lagrangian can be written as

$$\mathscr{L}=\tfrac{1}{2}\partial_\mu \mathbf{H}\cdot\partial^\mu \mathbf{H}-\tfrac{1}{2}m_{\mathrm{H}}^2\mathbf{H}\cdot\mathbf{H}+O(\mathbf{H}^4) \tag{3}$$

If $\mathbf{H}$ is related to an $SU(2)$ matrix in the usual way,

$$U=1+\frac{2i}{F}\boldsymbol{\tau}\cdot\mathbf{H}+\cdots=\exp\left(\frac{2i}{F}\boldsymbol{\tau}\cdot\mathbf{H}\right) \tag{4}$$

then substituting equation (4) into the Lagrangian

$$\mathscr{L}=\frac{1}{16}F^2\,\mathrm{Tr}(\partial_\mu U\cdot\partial^\mu U^\dagger)+\frac{1}{32E^2}\mathrm{Tr}[[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2]$$
$$+\frac{1}{8}m_H^2F^2(\mathrm{Tr}\,U-2) \tag{5}$$

we can obtain equation (3) (Adkins and Nappi, 1984a), where F is a dimensional constant, E is a dimensionless parameter, and m_{H} is the Higgs mass. In order to retain the chiral symmetry in equation (5), we drop the third term and obtain

$$\mathscr{L}=\frac{1}{16}F^2\,\mathrm{Tr}(\partial_\mu U\cdot\partial^\mu U^\dagger)+\frac{1}{32E^2}\mathrm{Tr}[[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2] \tag{6}$$

Equation (6) coincides with Skyrme's Lagrangian in form. It is easy to see that the Lagrangians (6) and (1) are the same in mathematical form, but they are not the same in physical meaning. In Skyrme's Lagrangian (1), $U=\exp[(2i/F)\boldsymbol{\tau}\cdot\boldsymbol{\pi}]$, $\boldsymbol{\pi}$ is the pion field, and there exists a strong interaction between pions. The interaction between Higgs is very weak in general. But the strength of the interaction does not determine in (6) whether the solitons can be formed. The difference of the parameters between equation (6) and Skyrme's Lagrangian (1) has no influence on whether the solutions of the solitons exist. Therefore, in spite of the fact that the fields and parameters in the Lagrangian given by equation (6) are different from those of equation (1), there exists the stable solution of the soliton. Substituting the Skyrme ansatz $U_0(x)=\exp[iF(\gamma)\boldsymbol{\tau}\cdot\mathbf{x}]$ into (6) and using the variational method, we get

$$[\tfrac{1}{4}\hat{\gamma}+2\sin^2F(\tilde{\gamma})]F''(\tilde{\gamma})+\tfrac{1}{2}\hat{\gamma}F'(\tilde{\gamma})+\sin 2F(\hat{\gamma})\cdot F'^2(\tilde{\gamma})$$
$$-\frac{1}{4}\sin 2F(\tilde{\gamma})-\frac{\sin^2F(\tilde{\gamma})\cdot\sin 2F(\tilde{\gamma})}{\tilde{\gamma}^2}=0 \tag{7}$$

where $\tilde{\gamma} = EF\gamma$ is a dimensionless variable. It is easy to see that equation (7) does not explicitly include any parameters. Therefore the numerical solution of equation (7) does not depend on the numerical value of E and F. Adkins *et al.* (1984a) obtained a numerical solution for the soliton of equation (7), that is the skyrmion. The skyrmion possesses the quantum numbers of spin $\frac{1}{2}$ and topological charge 1.

If the effect of Higgs particle masses ought not to be neglected, then we may use equation (5). Substituting the Skyrme ansatz into (5), we get

$$\begin{aligned}&[\tfrac{1}{4}\tilde{\gamma}^2 + 2\sin^2 F(\tilde{\gamma})]F''(\tilde{\gamma}) + \tfrac{1}{2}\tilde{\gamma}F'(\tilde{\gamma}) + \sin 2F(\hat{\gamma}) \cdot F'^2(\tilde{\gamma}) \\ &-\frac{1}{4}\sin 2F(\tilde{\gamma}) - \frac{\sin^2 F(\tilde{\gamma})}{\tilde{\gamma}^2} - \frac{1}{4}\beta^2\tilde{\gamma}^2 \sin F(\tilde{\gamma}) = 0 \qquad (8)\end{aligned}$$

where $\tilde{\gamma} = EF\gamma$, $\beta = m_{\mathrm{H}}/EF$. The $[-\frac{1}{4}\beta\tilde{\gamma}^2 \sin F(\tilde{\gamma})]$ term in equation (8) is a new term compared with equation (7). That is, equation (8) explicitly includes the dependence on the parameter β. Adkin and Nappi (1984b) have shown that there exist skyrmion solutions of equation (8) when $\beta = 0$ and $\beta = 0.263$. The case $\beta = 0$ is the case of equation (7). The $\beta = 0.263$ is the case of $F = 108$ MeV and $E = 4.84$, which are determined by the masses of the baryons. Because the mass of the skyrmion formed by the Higgs is not exactly known, we cannot determine exactly the value of β. So it is not very clear whether there exists an exact numerical skyrmion solution of (8). However, we may still expect that equation (8) has a skyrmion solution, because the numerical skyrmion solutions are not sensitive to β (Figure 1).

We may infer from the above-mentioned example, that there will exist another type of skyrmion so long as there exists a pseudoscalar triplet and

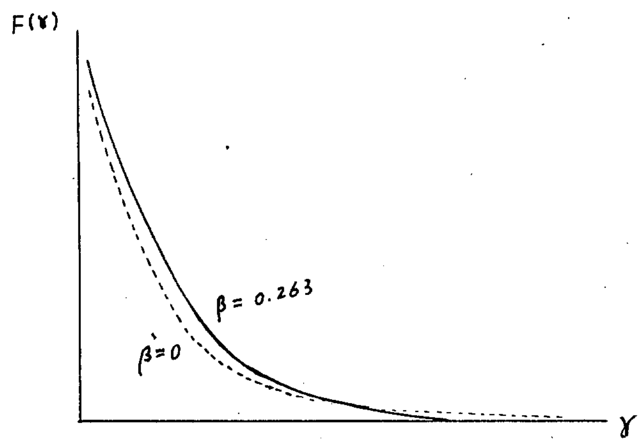

Fig. 1

we can construct a chiral symmetric Lagrangian of the Skyrme model. It is natural that the skyrmion of this type also possesses the quantum numbers of the spin $\frac{1}{2}$ and the topological charge 1.

Another question concerns the physical meaning of the skyrmion formed by the pseudoscalar Higgs triplet. Perhaps it is only a mathematical skyrmion. I explore this question here. In low-energy nuclear physics we observe a pion (pseudoscalar) interaction between nucleons. The solitons formed by these mesons can be interpreted as the nucleons in the Skyrme model. Similarly, if there exist other "meson" (pseudoscalar or scalar bosons) interactions between quarks or leptons, and if these "mesons" through some nonlinear interaction can form a skyrmion, then may this skyrmion be likewise interpreted as the quarks or leptons?

Now, scalar (or pseudoscalar) interaction between quarks or leptons are only mediated by Higgs. There exists only one physical Higgs on scalar φ_0 in the standard electroweak model. We do not know yet whether this physical Higgs scalar φ_0 may form a soliton. The nonstandard Higgs model may accommodate every variety of multiplet. In the two-Higgs-doublet model there may exist a physical pseudoscalar Higgs triplet $\mathbf{H}$ and a corresponding skyrmion. So we would like to present one problem. May the skyrmions formed by the pseudoscalar Higgs triplet be interpreted as the quarks or leptons?

If the skyrmions of this type may be interpreted as quarks or leptons, then the spin $\frac{1}{2}$ is consistent with the spin of the skyrmion, and the topological charge 1 may be defined as the quark number 1 (baryon number $\frac{1}{3}$) and the lepton number 1.

Up to here we have in part given the quantum numbers of the quarks or leptons using the skyrmion formed by the pseudoscalar Higgs triplet, and raised the possibility of interpreting this skyrmion formed by the pseudoscalar Higgs triplet as a quark or lepton. However, exact values of mass and three colors of the quarks have not yet been obtained in this paper (because the exact value of Higgs mass is not known). So we can not say that the skyrmion formed by the pseudoscalar Higgs triplet has been interpreted as a quark or lepton. A common problem of the aforementioned models is that the Higgs particles are all heavier than the quarks of the leptons. Although it is commonly accepted that Higgs particles are very heavy (they have not been discovered experimentally yet, but their masses relate to free parameters theoretically), the possibility that a light Higgs particle exists cannot be ruled out. For instance, Pham and Sutherland (1985) have shown a very small branching ratio for $K^+ \to \pi^+ + H$, so that a light Higgs particle is not excluded by this channel. If the light Higgs particle is confirmed by experiment in future, then the masses of skyrmions formed by such light Higgs particle will be not in conflict with the masses of the quarks. If the light Higgs particle is excluded by experiment in future, then a possible explanation is that solitons formed by Higgs particles are heavy quarks with masses about 1 TeV and above. These heavy quarks are the excited states of the solitons. When the excited states of solitons are transformed into ground states, they may be interpreted as ordinary quarks.

We discuss only the simplest Skyrme theory, that is, the case of the SU_2 chiral symmetry, where the Weiss-Zumino term is equal to zero. We may treat SU_3 or more complicated symmetry similarly.

The above example expresses a skyrmion cannot only be formed by π and σ mesons through some nonlinear interaction, but also by a pseudoscalar Higgs triplet. Of course the skyrmions of these two types are not the same in physical meaning. So we may propose some questions:

Is it possible that arbitrary mesons through some nonlinear interaction can always form the skyrmions of some type?

Is it possible that the skyrmions of some type can always be formed by some mesons through some nonlinear interaction? If they may form, then what are the physical meanings of the skyrmions of different types?

In summary:

1. So long as there exists a pseudoscalar triplet, for example, a pseudoscalar Higgs triplet, and one can construct a chiral symmetrical Lagrangian of the Skyrme model, then the existence of a skyrmion solution will be natural in this Lagrangian. These skyrmions are of different types, but their spins are all $\frac{1}{2}$ and their topological charges are all 1.

2. The physical meaning of skyrmions formed by a pseudoscalar Higgs triplet has not been interpreted satisfactorily. According to the naive analysis of the model in this paper, if these skyrmions are interpreted as quarks or leptons, then they have spin $\frac{1}{2}$, baryon number $\frac{1}{3}$, and lepton number 1, but cannot get the exact values of the masses and the three colors of the quarks. However, perhaps this is not too serious.

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